Application of Inverted Function to Alignment Optimization Design of Interchange Ramp

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Abstract. For adopting to the difficult terrain and restricted conditions, a probability of using the concept of inverted function into interchange linear to optimized design is built, which use the inverted function y=a/x as the interchange right-turn ramp linear. This function can be corresponded to the vehicle trajectory, decrease the area and cut the cost of interchange compared with the ordinary way"spiral-circle-spiral".By the analysis of how to use this function linear to be ramp, this paper get the way to set the ends of the ramp by circular curve or spiral curve. And this paper also take application example to illustrate the specific solution of how to use the inverted function to set the interchange right-turn ramp.

Introduction

Nowadays, the designs of interchange right-turn ramp alignment usually are "spiral-circle-spiral", and the change of radius of curvature on ramp is: $\infty \rightarrow R \rightarrow \infty$. The radius of curvature on spiral curve is changed continually but constant "R" on circle curve and this is not only a simple form and calculating way, but also can adapt to trajectory of car [1], but the triangle area which surrounded by the ramp curve and main line is bigger and the ramp curve is longer, so there will be some difficulties on the designing of ring-type and left-turn ramp in triangle area under the complex terrain or limited land[2,3]. The writer is trying to use the concept of inverted function into the optimal design of horizontal alignment by the establishment of inverted function calculation model of right-turn ramp, which considered the various designing reasons of ramp junction and tried to get optimal right-turn ramp horizontal alignment.

Basic Concept of Inverted Function

The inverted function in this article means basic inverted function $y=a/x(a>0)$ [4]. And the geometric characteristic of this inverted function in the first quadrant（$x>0,y>0$）is in Fig. 1

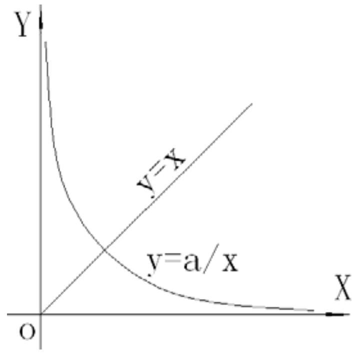

Fig. 1 Inverted function y=a/x(a>0,x>0,y>0) diagram

$$y = ax^{-1} \quad y^{'} = -ax^{-2} \quad y^{"} = 2ax^{-3} \cdot \tag{1}$$

The inverted function"$y = a/x \ (a > 0, x > 0, y > 0)$" has following geometric characteristics of linear:

It's continuous in the definite areas, and the n-order derivative is continuous;

Both of the "y=a/x" curvature and the rate of change are continuous;

"y=a/x" curvature radius changes continuous, it's ∞→R→∞. All of these natures are corresponded to the characteristics of vehicle trajectory[5,6].

Construction of right-turn ramp model

Construction of inverted function model

According to Formula (1), the writer can get the curvature radius function of y=a/x is:

$$R(x) = \frac{1}{\rho(x)} = \frac{[1 + y^{'2}]^{\frac{3}{2}}}{|y^{"}|} = \frac{(x^4 + a^2)^{\frac{3}{2}}}{2ax^3} \cdot \tag{2}$$

$R(x)$ first derivative is: $R^{'}(x) = \dfrac{3(x^4 + a^2)^{\frac{1}{2}}}{2a}(1 - \dfrac{a^2}{x^4})$

Analysis: when $x = \sqrt{a}$, $R^{'}(x) = 0$; when $x > \sqrt{a}$, $R^{'}(x) > 0$, $R(x)$ is in monotone increasing; when $x < \sqrt{a}$, $R^{'}(x) < 0$, $R(x)$ is in monotone decreasing. Only when $x = \sqrt{a}$, the curvature radius of curve is the minimum, $R(\sqrt{a}) \geq R_{\min} = R(\sqrt{a}) = \sqrt{2a}$,where $R_{\min}$ is the minimum radius without spiral curve designing[2].

It is can be satisfied ramp curve's minimum radius just by ensuring a's value. According to the general value and limits value of circle curve's minimum radius in the specification[7], this paper brings $R(\sqrt{a}) = \sqrt{2a}$ in to calculate a's value, and get the result as the following [Table 1]:

Table 1 Different design speed and curve radius of the circle corresponding to the value a

design speed in ramp(km/h)		80	70	60	50	40	35	30
minimum radius（m）	general value	280	210	150	100	60	40	30
	limits value	230	175	120	80	45	35	25
a value	general value	39200	22050	11250	5000	1800	800	450
	limits value	26450	15312.5	7200	3200	1012.5	612.5	321.5

Ramp junction setting

Function $y = a/x(a > 0)$, when $x \to 0$ and $x \to \infty$, radius of curve rate is ∞, in addition, $y = a/x(a > 0)$ is just approach with the asymptote x-axis & y-axis but without intersection. This paper needs to transfer the x-axis & y-axis to make the junction intersect with the axis. But after the moving, the slope of x-axis & y-axis in the point of intersection is zero or infinite, which can not connect with the curve slope(does not equal to zero). So this calculation needs to insert a circular curve or spiral curve to make the curve and line connect with each other easily, and the final result can be corresponded to the characteristics of vehicle trajectory[8,9].

When the radius inserting to the circular curve is longer than the smallest radius of the unsetting spiral curve, it can insert directly, and be calculated as the following steps:

Let $R(x)=\dfrac{(x^4+a^2)^{\frac{3}{2}}}{2ax^3}=R\geq R_{\min}$,

then: $$\begin{cases} x_1=a^{\frac{1}{3}}\{\dfrac{(2R)^{2/3}+\sqrt{(2R)^{4/3}-4a^{2/3}}}{2}\}^{\frac{1}{2}}, y_1=a/x_1 \\ x_2=a^{\frac{1}{3}}\{\dfrac{(2R)^{2/3}-\sqrt{(2R)^{4/3}-4a^{2/3}}}{2}\}^{\frac{1}{2}}, y_2=a/x_2 \end{cases}. \quad (3)$$

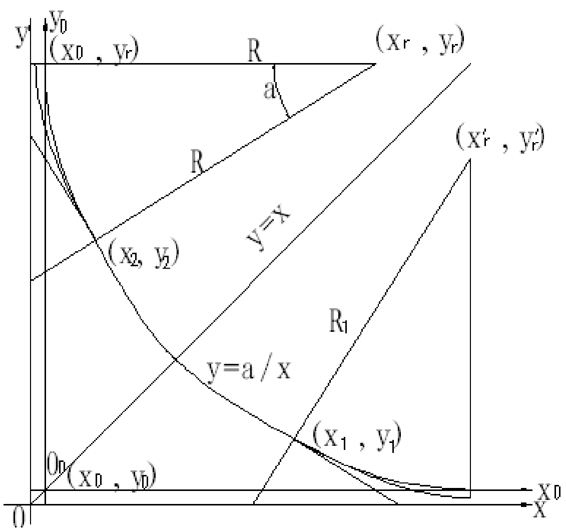

Fig. 2 Insert a circular curves

The coordinates in Fig. 2 are under X**O**Y axes. Then need to cut X_0 and Y_0 to transfer it to the coordinates under X_0**O**Y_0 axes. The slope of the curve $y=a/x(a>0)$ in the inserting point is: $k=y'=-a/x^2$, and the slope in the point (x_2, y_2) which is perpendicular to the curve's radius is: $k_R=-/k_2=x_2{}^2/a$, then:

$$\begin{cases} x_r=x_2+Rcon\partial=x_2+R\dfrac{1}{\sqrt{k_r^2+1}}=x_2+R\dfrac{a}{\sqrt{a^2+x_2^2}} \\ y_r=y_2+R\sin\partial=y_2+R\dfrac{k_r}{\sqrt{k_r^2+1}}=y_2+R\dfrac{x_2^2}{\sqrt{a^2+x_2^2}} \end{cases}. \quad (4)$$

the offset distance from y-axis to axis y_0: $x_o=x_r-R=x_2+(\dfrac{a}{\sqrt{a^2+x_2^2}}-1)$. (5)

In the same way, calculate the offset distance of x_r', y_r' and x-axis is $y_o=y_r'-R_1$, and transfer

$y = a/x(a > 0)$ to $y = a/(x - x_0) + y_0(a > 0)$ by the offset of the axes. Then the function of the whole ramp is:

$$y = \begin{cases} y_r - y_0 + \sqrt{2Rx - x^2} & 0 < x \le x_2 - x_0 \\ a/(x - x_0) + y_0 & x_2 - x_0 \le x \le x_1 - x_0 \\ R - \sqrt{R^2 - (x - x_r^{'} + x_0)^2} & x_1 - x_0 \le x \le x_r^{'} - x_0 \end{cases} \cdot \quad (6)$$

Of course, insert circular curves in same of different radius at both of the 2 ends according to the terrain' differences and the setting qualification. When there are circular curves with the same R，the whole ramp curve is symmetrical corresponding to x=y. And according to the symmetry, $x_0 = y_0$, $x_2 = y_1$, $x_1 = y_2$, $x_r^{'} = y_r$, $x_r = y_r^{'}$, then get the location and parameters of the curve only by calculating on end of the curve x_0, x_r, y_r, or calculate the two ends separately when the radius are different. From the qualitative analysis of Fig.2 , the bigger radius of the inserting circular curve, the longer of the ramp curve will be. When the length of the ramp curve is too long, the range of the ramp in line is so big which is parallel with the line almost, it's uneconomical obviously. Because the vehicle speed usually slower than design speed, then choose $R=R_{min}$ or $R > R_{min}$.

Key points of using circular curve to set the end: 1）choose "a" according to Fig.1 after the confirmation of design speed; 2）Calculate the location of inserting the circular curve according to the superelevated smallest radius; 3）Calculate the offset of x-axis & y-axis; 4）Write the junction of the ramp; 5）"R"of $y = a/x(a > 0)$ of the two ends of the circular curve can be same or different according to the practical demand.

Merits of the Model

Compared with circle- spiral -circle system, the length of inverted function curve is shorter when there are same circular curve radius and inverted function minimum radius. And also in this situation, the ramp area will smaller, the interchange range will decrease and there will be lower cost, just as Fig.3.

When there are the same inserting point on circular curve and inverted function what means the range of interchange on line is unchanged, the triangle area surrounded by inverted function is bigger and the ramp's area is smaller when there are same smallest radius of circular curve and inverted function. This way can supply more convenient to set other facilities, ring-type ramp as Fig.4:

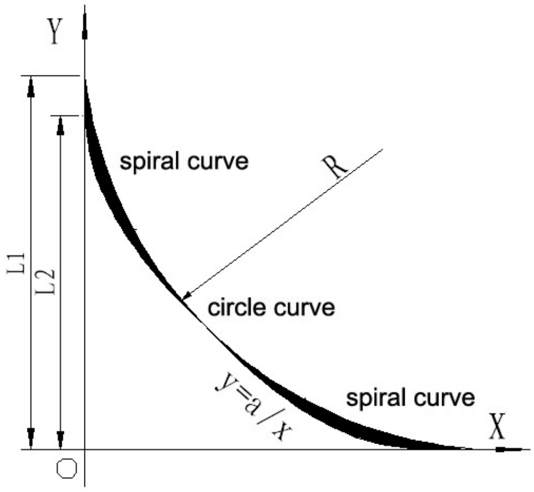

Fig. 3 Inverted curve radius of the circle as the same as the smallest radius

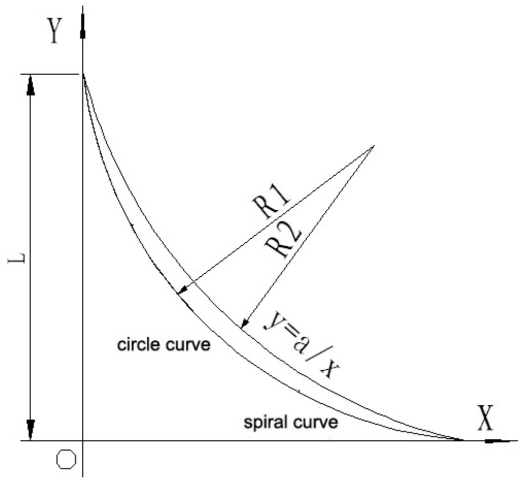

Fig. 4 Inverted curve radius of the circle as the same as the insert point

Application example

This paper will take B ramp of the right-turn ramp of one city's interchange as an example to examine the engineering applications of the right-turn ramp which designed by $y = a / x$. Designed speed of main line 1 and main line 2 is 60Km/h, the 2 lines orthogonal, and the directions of coordinates system's axes and the interchange's line are same.

As the specification'recommendation, designed speed of right-turn ramp is 30-40Km/h，but because right-turn ramp should use the up-limitation and the middle, so need 40Km/h.

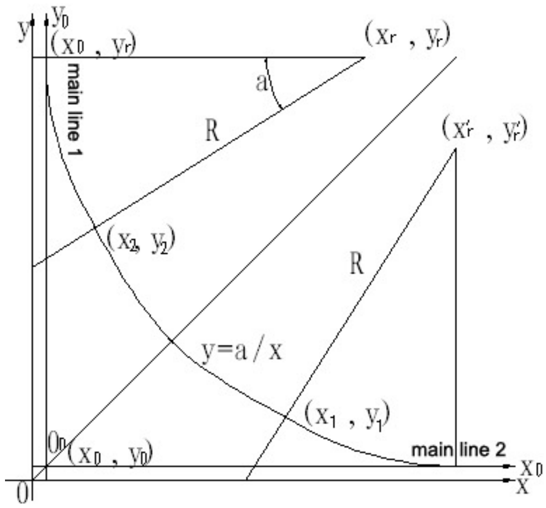

Fig. 5 Schematic ramp

Choose a and R

According to Fig.1, the general value of a is 1800 and the limited value is 1012.5, need a=2000. As the specification, don't design spiral curve's minimum radius R_{min}=600m, both ends of the ramp get the same radius R=600m.

According to formula（3）, calculating the inserting point which links circle curve:

$$\begin{cases} x_1 = 133.0446,\ y_1 = 15.0325 \\ x_2 = 15.0325,\ y_2 = 133.0446 \end{cases}$$

According to formula（4）, calculating $x_r, y_r, y_r^1, \partial$ and offset x_0, y_0 under the coordinates of XOY:

$$\begin{cases} x_r = 611.2388 \\ y_r = 200.4099 \end{cases}$$

According to the symmetry, $y_r^{'} = x_r = 611.2388$

$$k_R = x_2^{\ 2} / a = 15.0325^2 / 2000 = 0.1130$$
$$\partial = tg^{-1} k_R = 6.4464^o$$

According to the formula (5) get offset:

$$\begin{cases} x_0 = 11.2388 \\ y_0 = 11.2388 \end{cases}$$

Function formula under the coordinates of X_0OY_0 and according to formula（6）:

$$y = \begin{cases} 189.1711 + \sqrt{1200x - x^2} & 0 < x \le 3.7937 \\ \dfrac{2000}{x - 11.2388} + 11.1388 & 3.7937 \le x \le 121.8058 \\ 600 - \sqrt{600^2 - (x - 189.1711)^2} & 121.8058 \le x \le 600 \end{cases}$$

Calculating length of ramp:

Length of circle curve length: $l_y = \pi R\partial / 180 = 67.5064m$,Length of function $y = a / x$ is $l_a = \int_a^b \sqrt{1 + y'^2} dx = \int_{3.7937}^{121.8058} \sqrt{1 + y'^2} dx =$ 94.204 m ,length of the whole ramp is $l = 2l_y + l_a = 229.2170\, m$.

Conclusion

Using inverted function to design interchange right-turn ramp is a flexible way. It doesn't need to establish a independent mathematic model to every curve combination, but only need to change the parameter value of every key element in the unity mathematic model and to calculate them. Also, this design can be in accordance with the characteristics of vehicle trajectory, decreasing interchange's range, shorter ramp's length, and the calculating process just needs elementary function. Lastly, this design will be effective when there are limitations of terrain or other conditions.

Acknowledgements

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References

[1] Yang Shao-wei: *Road survey and design* (China Communications Press ,Beijing 2004).

[2] Yang Shao-wei: *Road interchange Planning and Design* (China Communications Press, Beijing 2000).

[3] Sui Yong-qin, Zhang Xue-hua and Wang Jia-hong: *Calculation of Incomplete Transition Curve in Interchange Ramp*, Central South Highway Engineering 28(3):53-55 (2003).

[4] Zhang Yuan: Study on Freeway On-Ramp Control, submitted to Chang'an University(2005).

[5] Ye Yong-gang: Freeway on-ramp neural network control, submitted to Harbin Institute of Technology(2006).

[6] Zhou Lin: *Application of Fuzzy Mathematics to Alignment Optimization Design of Interchange Ramp*, Journal of Chongqing Jiao Tong University(Natural Science) 26(5):53-55 (2007).

[7] Zhang Hao-ping, Pen Li and Hu Hu: *Application of Cubic Curve to Exit and Entrance at Interchange*, Central South Highway Engineering 28(3):29-31 (2003).

[8] Zhang Yu-mei, Qu Shi-ru and Wen Kai-ge: *Improving the Algorithm for On-Ramp Control Based on BP Neural Networks*, Journal of Northwestern Polytechnical University 25(6):820-823 (2007).

[9] Ma Su, Zhang Yu-mei: *An Overview of Freeway On-ramp Control Algorithms*, Computer Simulation 26(1):271-275 (2009).